

Es symmetrisch

Symmetrisch

$$\sigma = \{\lambda_1(k_1), \lambda_2(k_2), \dots, \lambda_n(k_n)\}$$

$$\dim V(\lambda_i) = d_i$$

$$\sum d_i = \sum k_i = n$$

$$\exists P \text{ matrixe} / D = P^{-1} \cdot A \cdot P$$

Diagonalform

Diagonalform

$\exists P_{\text{orth}}$ matrixe orthogonal

$$D = (P_{\text{orth}})^T \cdot A \cdot P_{\text{orth}}$$

Diagonalform
orthogonal

$$A \cdot x - \lambda I x = (0)$$

$$A \cdot x = \lambda x / (A - \lambda I) = 0 \rightarrow \lambda \text{ autobalioak}$$

x_i autobektoreak

$$P = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$\sigma(A) = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n\}$$

spektroa ∇ Anmerkungen: nicht
Kontinuum heißt diskret

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_n \end{pmatrix}$$

$$V \left\{ \begin{array}{l} \mathbb{R}^n ; n \in \mathbb{R} \\ \mathcal{P}_n ; \forall v \in \mathcal{P}_n \\ \mathcal{M}_{m,n}(\mathbb{R}) \\ C[a,b] \end{array} \right.$$

$\forall +, \cdot \rightarrow$ Länge kompositen legen

Basis kompositen legen

$$\rightarrow \{(1, 1, 0), (3, 2, 0), (2, 3, 0), \dots\}$$

Bektoreen, Kombinationen
linearer Bektoren, lineare
lineare Substruktur.
Itasuria multibektore

$$\text{Aspektu bektoreak} \leftarrow S = \mathcal{L}(F) = \mathcal{L}(B_S) \rightarrow \text{Sren oinarria}$$

$$h(B_S) = h = \dim S$$

sortutako lotura

sortutako askeri

$$\dim S \leq \dim V$$

\hookrightarrow V-ren aspektu propioa da

$$\dim S = \dim V$$

$$\hookrightarrow S = V$$

$$\dim \geq 1$$

\hookrightarrow Dimension bat? Inongo da harritzaile bat da

bektore lineari
independenteak \leftarrow Heine $\left\{ \begin{array}{l} h = m \text{ askeri} \\ h < m \text{ lotura} \end{array} \right.$
kopurua m bektore-kopurua

Oinarri bektore
heineak:
bektoreak, zehazki
beheko
heine bektore

$$\|\vec{x} - \vec{y}\| = d(\vec{x}, \vec{y}) = \sqrt{\langle \vec{x} - \vec{y}, \vec{x} - \vec{y} \rangle}$$

B: bektoreen arteko angelua

$$\cos \theta = \frac{\langle \vec{x}, \vec{y} \rangle}{\|\vec{x}\| \cdot \|\vec{y}\|}$$

B: bektoreen arteko distantzia

$$\begin{aligned} \vec{x} &= (1, 2, 3) \\ \vec{y} &= (2, 1, 4) \rightarrow \vec{x} \cdot \vec{y} = (-1, 1, 1) \\ d(\vec{x}, \vec{y}) &= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \end{aligned}$$

bektoreen $\left(\begin{matrix} V \\ \langle \cdot, \cdot \rangle \end{matrix} \right)$

Bektore baten norma kalkulatu

$$\|\vec{x}\| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$$

$$\begin{aligned} \vec{x} &= (1, 2, 3) \\ \|\vec{x}\| &= \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \\ \vec{y} &= (2, 1, 4) \\ \|\vec{y}\| &= \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21} \end{aligned}$$

bideketak eskalarra

Ornari baten ortogonalaketa

Hurbilketa lineal hobea

Bektore baten proiektzio beste bektore baten gainean

B: bektoreen arteko angelua

$$P_{\vec{y}} \vec{x} = \frac{\langle \vec{x}, \vec{y} \rangle}{\|\vec{y}\|^2} \vec{y}$$

$$P_{\vec{x}} \vec{y} = \frac{\langle \vec{y}, \vec{x} \rangle}{\|\vec{x}\|^2} \vec{x}$$

$$P_{\vec{y}} \vec{x} = \frac{16}{21} (2, 1, 4) = \left(\frac{32}{21}, \frac{16}{21}, \frac{64}{21} \right)$$

$$P_{\vec{x}} \vec{y} = \frac{16}{14} (1, 2, 3) = \left(\frac{8}{7}, \frac{16}{7}, \frac{24}{7} \right)$$

Gram-Schmidt proiektzio

$$B_S = (\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$$

$$B_{OG} = (\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n)$$

$$\mathcal{L}(B_S) = S$$

$$\hookrightarrow \mathcal{L}(B_{OG}) = S$$

Bektoreak bideketan, elkarren arteko bideketak eskalarrek zero emango du.

$$\vec{v}_1 = \vec{u}_1 \quad \vec{v}_2 = \vec{u}_2 - \frac{\langle \vec{u}_2, \vec{v}_1 \rangle}{\|\vec{v}_1\|^2} \vec{v}_1$$

$$\vec{v}_3 = \vec{u}_3 - \frac{\langle \vec{u}_3, \vec{v}_1 \rangle}{\|\vec{v}_1\|^2} \vec{v}_1 - \frac{\langle \vec{u}_3, \vec{v}_2 \rangle}{\|\vec{v}_2\|^2} \vec{v}_2$$

$$\vec{v}_n = \vec{u}_n - P_{\vec{v}_1} \vec{u}_n - P_{\vec{v}_2} \vec{u}_n - \dots - P_{\vec{v}_{n-1}} \vec{u}_n$$

ELS BATERAEZINA: SOLUTIO HURBILKETA

- 1) $B_S \rightarrow$ oinarri bat lortu
- 2) $B_S \rightarrow B_{OG}$ (oinarri ortogonalak)
- 3) \vec{b} -ren HLU(s) = \vec{b}^*
- 4) $A \cdot \vec{x} = \vec{b}^* \rightarrow$ Bateria garra
- 5) Sistema ebaketa, \vec{x} : sol hurbilketa

DIAGONALKETA

- 1) $|A - \lambda I| = 0$
- 2) Autobektoreak \rightarrow espektro $\rightarrow D$
Autobektoreak $\rightarrow P$
- 3) $P^{-1}, P_{01}, (P_{02})^T, \dots$

Bektoreen moduluak \uparrow daturak ortogonalaketa

Ortonormalizazioa

$$\left\{ \frac{\vec{v}_1}{\|\vec{v}_1\|}, \frac{\vec{v}_2}{\|\vec{v}_2\|}, \frac{\vec{v}_3}{\|\vec{v}_3\|}, \dots \right\}$$

ELS BATERIA D'UNA SOLUCIÓ HORBJLOVA

$$x_1 + x_3 = 1$$

$$x_2 + 2x_3 = 1$$

$$x_1 + 2x_2 + 2x_3 = 1$$

$$x_1 + x_2 + 3x_3 = 1$$

$$\Delta_2 = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 1 \end{pmatrix}$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $\bar{v}_1 \quad \bar{v}_2 \quad \bar{v}_3 \quad \bar{b}$

$$B_2 = \{(1,0,1,1), (0,1,2,1), (1,2,2,1)\}$$

$$h(B_2) = 3 \text{ - batreu kepuen}$$

↓
Omnibus da.

$$B_{00} = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$$

$$\bar{v}_1 = \bar{v}_1 = (1,0,1,1)$$

$$\|\bar{v}_1\|^2 = 3 \quad \langle \bar{v}_2, \bar{v}_1 \rangle = 3$$

$$\|\bar{v}_2\|^2 = 3 \quad \langle \bar{v}_3, \bar{v}_1 \rangle = 6$$

$$\langle \bar{v}_3, \bar{v}_2 \rangle = 3$$

$$\bar{v}_2 = \bar{v}_2 - \frac{\langle \bar{v}_2, \bar{v}_1 \rangle}{\|\bar{v}_1\|^2} \bar{v}_1 = (0,1,2,1) - \frac{3}{3}(1,0,1,1) = (-1,1,1,0)$$

$$\bar{v}_3 = \bar{v}_3 - \frac{\langle \bar{v}_3, \bar{v}_1 \rangle}{\|\bar{v}_1\|^2} \bar{v}_1 - \frac{\langle \bar{v}_3, \bar{v}_2 \rangle}{\|\bar{v}_2\|^2} \bar{v}_2 = (1,2,2,3) - \frac{6}{3}(1,0,1,1) - \frac{3}{3}(0,1,1,0) = (0,1,-1,1)$$

$$B_{00} = \{(1,0,1,1), (-1,1,1,0), (0,1,-1,1)\}$$

$$\bar{b} = \sum_{i=1}^3 \frac{\langle \bar{b}, \bar{v}_i \rangle}{\|\bar{v}_i\|^2} \bar{v}_i = \frac{\langle \bar{b}, \bar{v}_1 \rangle}{\|\bar{v}_1\|^2} \bar{v}_1 + \frac{\langle \bar{b}, \bar{v}_2 \rangle}{\|\bar{v}_2\|^2} \bar{v}_2 + \frac{\langle \bar{b}, \bar{v}_3 \rangle}{\|\bar{v}_3\|^2} \bar{v}_3 = \frac{3}{3} \bar{v}_1 + \frac{1}{3} \bar{v}_2 + \frac{1}{3} \bar{v}_3 = (2/3, 2/3, 1, 4/3)$$

$$\langle \bar{b}, \bar{v}_1 \rangle = 3 \quad \|\bar{v}_1\|^2 = 3$$

$$\langle \bar{b}, \bar{v}_2 \rangle = 1 \quad \|\bar{v}_2\|^2 = 3$$

$$\langle \bar{b}, \bar{v}_3 \rangle = 1 \quad \|\bar{v}_3\|^2 = 3$$

$$\Delta \bar{x} = \bar{b} \Rightarrow \Delta \bar{x} = \bar{b}$$

$$\Delta_1' = \begin{pmatrix} 1 & 0 & 1 & 4/3 \\ 0 & 1 & 2 & 2/3 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 4/3 \end{pmatrix} \xrightarrow{R_3 - R_1, R_4 - R_1} \begin{pmatrix} 1 & 0 & 1 & 4/3 \\ 0 & 1 & 2 & 2/3 \\ 0 & 2 & 1 & -1/3 \\ 0 & 1 & 2 & 4/3 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_4} \begin{pmatrix} 1 & 0 & 1 & 2/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 0 & 3 & -3/3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 4/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h(\Delta_1) = h(\Delta_1') = 3$$

$$x_1 = -1/3$$

$$x_2 = -4/3$$

$$x_3 = 1$$

$$e = d(\bar{b}, \bar{b}') = \|\bar{b} - \bar{b}'\| = \sqrt{(1/3)^2 + (1/3)^2 + (1/3)^2} = \sqrt{1/3}$$

$$B = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \quad |B - \lambda I| = 0 \quad \begin{vmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{vmatrix} = (2-\lambda)(-2-\lambda) + 3 = 0 \Rightarrow -4 - 2\lambda + 2\lambda + \lambda^2 + 3 = 0 \Rightarrow \lambda^2 - 1 = 0$$

$$\lambda_1 = 1 \quad (k=1)$$

$$\lambda_2 = -1 \quad (k=1)$$

$$\sigma(B) = \{1, -1\} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{\lambda = 1}$$

$$\begin{pmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{pmatrix} \Rightarrow \begin{pmatrix} 2-1 & -3 \\ 1 & -2-1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & 0 \\ 1 & -3 & 0 \end{pmatrix} \quad x_1 - 3x_2 = 0 \rightarrow x_1 = 3x_2 \rightarrow (x_1, x_2) = (3x_2, x_2) = x_2(3, 1)$$

$$V(1) = \mathcal{L}\{(3, 1)\}$$

$$\boxed{\lambda = -1}$$

$$\begin{pmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{pmatrix} \Rightarrow \begin{pmatrix} 2+1 & -3 \\ 1 & -2+1 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & -3 \\ 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & -3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & 0 \\ 1 & -1 & 0 \end{pmatrix} \quad x_1 - x_2 = 0 \rightarrow (x_1, x_2) = (x_2, x_2) = x_2(1, 1)$$

$$V(-1) = \mathcal{L}\{(1, 1)\}$$

$$P = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$

$$D = P^{-1} B P \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$